Circles, Angle Measures and Arcs

A.5C	Use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions
A.6G	Relate direct variation to linear functions and solve problems involving proportional change.
A.7A	Analyze situations involving linear functions and formulate linear equations or inequalities to solve problems;
G.2A	Use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships.
G.2B	Make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.
G.3D	Use inductive reasoning to formulate a conjecture.
G.9C	Formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models.

Materials

Advance Preparation:

- Student access to computers with Geometer's Sketchpad and necessary sketches and/or a projection device to use Geometer's Sketchpad as a class demonstration tool.

For each student:

- Graphing calculator
- **Create an "Arc Measuring Tool"** activity sheet
- **Angles Formed by Chords Intersecting Inside a Circle** activity sheet
- **Angles Formed by Secants Intersecting Outside a Circle** activity sheet
- **Other Intersecting Lines and Segments** activity sheet
- **Quad-Tri Incorporated** activity sheet

For each student group of 3 - 4 students:

- Compasses
- Protractors
- Patty paper or tracing paper
- Rulers
- Scissors

ENGAGE

The Engage portion of the lesson is designed to create student interest in the relationships among the measures of angles formed by segments in circles and related arc measures. This part of the lesson is designed for groups of three to four students.

1. Distribute two sheets of patty paper, a compass, ruler, protractor and a pair of scissors to each student.
2. Prompt students to use a compass to construct a large circle on one sheet of patty paper. Then have them construct a second circle, congruent to the first circle on the second sheet of patty paper.
3. Distribute the **Create an Arc Measuring Tool** activity sheet. Students should follow the directions on the sheet.
4. On their second circle, students should draw two intersecting chords that do not intersect in the center of the circle.
5. Students should use the available measuring tools to find angle measures and estimate arc measures.
6. Students will record their individual results, share results with their group, and discuss observations.
7. Debrief the activity using the Facilitation Questions.

Facilitation Questions – Engage Phase

1. When you fold a diameter, how many degrees are in each semi-circle?
   180° semi means half; one-half of 360° is 180°.
2. When you fold a second diameter perpendicular to the first, how many degrees are in each quarter-circle?
   90° one quarter means one-forth, one-forth of 360° is 90°.
3. How can you make your "Arc Measuring Tool" a more precise measuring tool? *By continuing the folding process you can have 45°, 22.5° etc.*
4. How did you use your "Arc Measuring Tool" to estimate the measures of the arcs in your circle? *Answers may vary. Students should be able to explain how they used known "benchmarks" like 90°.*
5. What other method could you use to determine the measures of the arcs on your second circle?
   Answers may vary. Students should realize they can draw central angles that intercept the arc they are trying to measure and the measure of the central angle is equal to the measure of the intercepted arc.
6. What similarities do your measurements have with measurements taken by other members of your group?
   Answers may vary. Students may notice, vertical angles are congruent; the sum of the measures of all arcs of the circle is 360° etc.
7. How can you determine if your observations will be true for any circle?
   Answers may vary. Students should realize that data for several circles could be collected and analyzed to verify conjectures.

EXPLORE

The Explore portion of the lesson provides the student with an opportunity to participate actively in the exploration of the mathematical concepts addressed. This part of the lesson is designed for groups of three to four students.

1. Distribute the **Angles Formed by Chords Intersecting Inside a Circle** activity sheet.
2. Students should open the sketch **Twochords-in.**
3. Have students follow the directions on the activity sheet to collect data and explore the relationship between angle measures and intercepted arcs.
   Note: If students are not familiar with the operation of Geometer's Sketchpad, they will need the necessary instruction at this time.

Facilitation Questions – Explore Phase

1. What patterns do you notice in the table?
   Students should notice that relationships such as vertical angles are equal or the sum of the measures of the arcs is twice the measure of the angles, etc.

2. Where do you see proportional relationships in your table?
   Properties of proportional relationships can be explored at this time. Remind students of scale factors and constant of proportionality.

3. How did you use your table to develop an algebraic rule for this relationship?
   Answers may vary. Students may have used the process column, constant of proportionality, finite differences, etc.

EXPLAIN

The teacher directs the Explain portion of the lesson to allow the students to formalize their understanding of the TEKS addressed in the lesson.

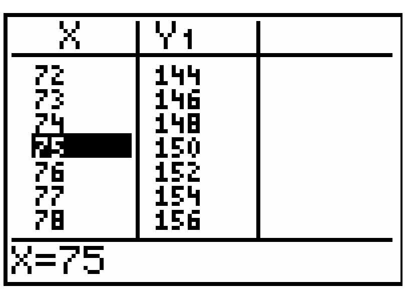
1. Debrief the **Angles Formed by Chords Intersecting Inside a Circle** activity sheet. Use the Facilitation Questions to help students make connections among methods that can be used to calculate the measure of the angle or intercepted arc.
2. Have each student group present the way they found the algebraic rule and give a verbal description of the relationship.
3. Be sure students understand how to use the Geometer's Sketchpad sketches.

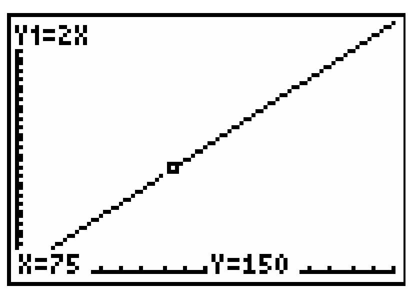
Facilitation Questions – Explain Phase

1. What is the meaning of your algebraic rule in this relationship?
   Two times the angle measure equals the sum of the intercepted arcs.
2. If you know the measure of the angle, how can you find the sum of the measures of the intercepted arcs?
   Multiply the angle measure by 2.
3. If you know the measure of each intercepted arc, how can you find the angle measure?
   Find the sum of the arcs and then divide by 2.
4. If you know the measure of one angle and one intercepted arc, how could you find the measure of the other intercepted arc?
   Double the angle measure then subtract the known arc from that value.
5. If you know the measure of one angle and one intercepted arc, what algebraic equation could you write to calculate the measure of the other intercepted arc?

$$2(angle) = arc1 + arc2$$

6. How could you use the table or graph feature of your graphing calculator to determine the measure of an angle formed by two intersecting chords if the measures of its intercepted arcs are 30° and 120°?





ELABORATE

The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS to a new situation. This part of the lesson is designed for groups of three to four students.

1. Distribute the **Angles Formed by Secants Intersecting Outside a Circle** activity sheet.
2. Students should open the sketch **Twosecants-out.**
3. Have students follow the directions on the activity sheet to collect data and explore the relationship between angle measures and intercepted arcs.
4. Debrief the **Angles Formed by Secants Intersecting Outside a Circle** activity sheet.
5. Distribute the **Other Intersecting Lines and Segments** activity sheet.
6. Prompt students to open the sketches as directed and explore the relationships.
7. Debrief the **Other Intersecting Lines and Segments** activity sheet.

Facilitation Questions – Elaborate Phase

1. What patterns do you notice in the table?
   Students should notice that relationships such as vertical angles are equal or the sum of the measures of the arcs is twice the measure of the angles etc.
2. Where do you see proportional relationships in your table?
   Properties of proportional relationships can be explored at this time.
   Remind students of Scale factors and constant of proportionality.
3. How did you use your table to develop an algebraic rule for this relationship?
   Answers may vary. Students may have used the process column, constant of proportionality, finite differences etc.
   After completing the summary table for this activity, what general statements can you make about angles formed by lines and segments that intersect circles?

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.

1. Distribute the Mathematics Chart.
2. Provide each student with the **Quad-Tri Incorporated** activity sheet.
3. Upon completion of the activity sheet, a rubric should be used to assess student understanding of the concepts addressed in the lesson.

Answers and Error Analysis for selected response questions:

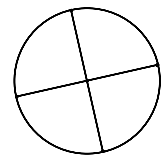
Question Number	*TEKS*	*Correct Answer*	*Conceptual Error*	*Conceptual Error*	*Procedural Error*	*Procedural Error*	*Guess*
1	G.9(c)	D	B	C	A		
2	G.9(c)	D	B	C	A		
3	G.9(c)	A	C	D	B		
4	G.9(c)	A	C	B	D		

Create an "Arc Measuring Tool"

1. You should have two sheets of Patty Paper. On each sheet construct a large circle. Be sure your circles are congruent to each other.

2. Cut out each circle and set one aside.

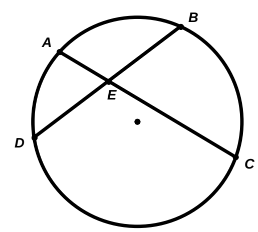
3. Fold a diameter in the second circle. Unfold the circle then fold a second diameter perpendicular to the first diameter. You should have something that looks like this.



4. What special point is the point of intersection of the diameters? How do you know? *The point is the center of the circle. It is the midpoint of the diameters so it must be the center.*

5. You now have a tool to estimate the number of degrees in arcs of your other circle. How can you make your "Arc Measuring Tool" a more precise measuring tool? *By continuing the folding process you can have 45°, 22.5° etc.*

6. In your second circle, use a straight edge to draw two chords that intersect at a point that is not the center of the circle. Label your diagram as shown. Then use your available tools to find or estimate the necessary measures to complete the table below.



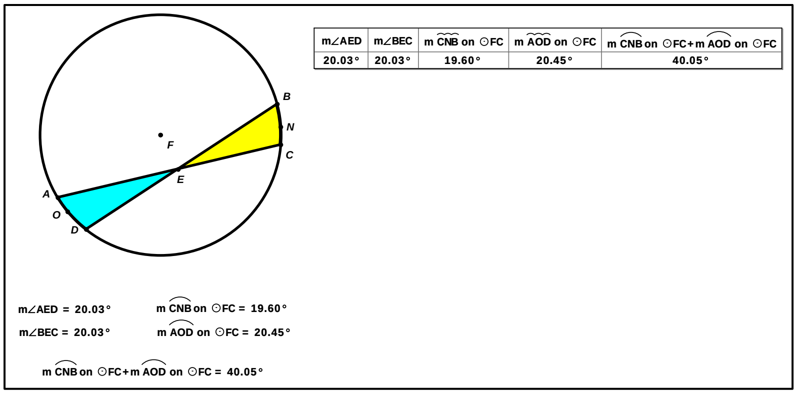
7. Record your name, your measurements and the name of each member of your group along with their measurements in the table.

Name	$m\angle AED$	$m\angle BEC$	$m\overset{\frown}{BC}$	$m\overset{\frown}{AD}$
	50°	*50°*	*60°*	*140°*
	65°	*65°*	*70°*	*60°*
	43°	*43°*	*40°*	*46°*
	124°	*124°*	*82°*	*166°*
	130°	*130°*	*100°*	*160°*

8. What patterns do you observe in the table? *Answers may vary. Students should observe that the* $m\angle AED = m\angle BEC$ *or that the sum of the measures of the arcs is twice the measure of each angle.*

Angles Formed by Chords Intersecting Inside a Circle

Open the sketch **Twochords-in.**



1. **Double click on the table to add another row, then click and drag point *B* away from point *N*. What do you observe?**
   The measures change.
2. **Double click on the table again, and then move point *C* away from point *N*. Be sure point *N* stays between *B* and *C*.**
3. **Double click again, but this time drag point A away from point *O*. Double click again and drag point *D* away from point *O*. Be sure point *O* stays between *A* and *D*.**
4. **Be sure you have some small angle measures that are greater than 0° and some large angle measures that are less than 180°. Repeat this process until you have 10 rows in your table.**
5. **Record the data from the computer in the table below.**

$m\angle AED$	$m\angle BEC$	$m\overset{\frown}{BC}$	$m\overset{\frown}{AD}$	$m\overset{\frown}{CNB} + m\overset{\frown}{AOD}$
20.03	*20.03*	*19.60*	*20.45*	*40.05*
35.53	*35.53*	*50.61*	*20.45*	*71.06*
42.57	*42.57*	*64.69*	*20.45*	*85.14*
56.60	*56.60*	*64.69*	*48.51*	*113.20*
68.98	*68.98*	*64.69*	*73.28*	*137.97*
79.68	*79.68*	*86.09*	*73.28*	*159.37*
96.54	*96.54*	*119.79*	*73.28*	*193.07*
125.02	*125.02*	*119.79*	*130.24*	*250.03*
144.07	*144.07*	*119.79*	*168.35*	*288.14*
170.00	*170.00*	*171.65*	*168.35*	*340.00*

6. **What patterns do you observe in the table?**
Answers may vary. Students should observe the $m\angle AED = m\angle BEC$ *and the sum of the measures of the arcs is twice the measure of each angle.*

7. **To explore the relationship between the sum of the measures of the intercepted arcs and the measure of** $\angle AED$**, transfer the necessary data from the table in question 3 to the table below.**

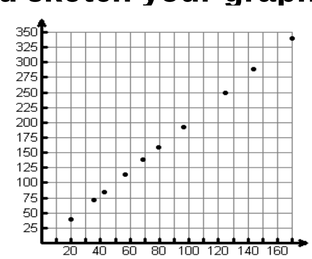
$m\angle AED$ (x)	PROCESS	$m\widehat{CNB} + m\widehat{AOD}$ (y)
20.03	*(2) 20.03*	*40.05*
35.53	*(2) 35.53*	*71.06*
42.57	*(2) 42.57*	*85.14*
56.60	*(2) 56.60*	*113.20*
68.98	*(2) 68.98*	*137.97*
79.68	*(2) 79.68*	*159.37*
96.54	*(2) 96.54*	*193.07*
125.02	*(2) 125.02*	*250.03*
144.07	*(2) 144.07*	*288.14*
170.00	*(2) 170.00*	*340.00*
x	$2x$	y

8. **Use the process column to develop an algebraic rule that describes this relationship.**
$y= 2x$

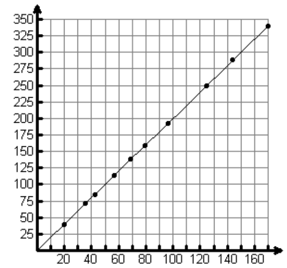
9. **Write a verbal description of the relationship between the sum of the measures of the intercepted arcs and the measure of the angle formed by the intersecting chords.**
Two times the measure of the angle is equal to the sum of the measures of the intercepted arcs. The sum of the measures of the intercepted arcs divided by 2 is equal to the measure of the angle.

10. **Create a scatterplot of the sum of the arc measures versus angle measure. Describe your viewing window and sketch your graph.**

x-min = *0*
x-max = *170*
y-min = *0*
y-max = 350



11. Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.



12. Does the graph verify your function rule? Why or why not?
Yes. The graph of the function rule passes through each data point.

13. What is the measure of an angle formed by two intersecting chords if the measures of its intercepted arcs are 30° and 120°?
75°

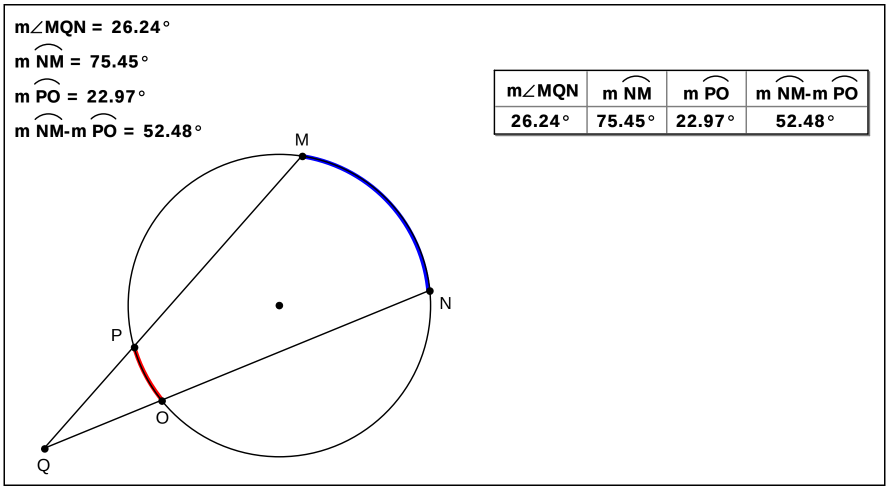
14. What is the sum of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting chords is 56°?
112°

15. Make a general statement about how you can determine the measure of an angle formed by two intersecting chords when you know the measures of the intercepted arcs.
To determine the measure of the angle, add the two intercepted arcs then divide by 2.

16. Make a general statement about how you can determine the sum of the measures of the intercepted arcs when you know the measure of the angle formed by two intersecting chords.
To determine the sum of the measures of the intercepted arcs, multiply the measure of the angle by 2

Angles Formed by Secants Intersecting Outside a Circle

Open the sketch **Twosecants-out**.



1. **Double click on the table to add another row, then click and drag point *M*. What do you observe?**
   The measures change.
2. **Double click on the table to add another row, and then move point *M* again. Double click again, but this time drag point N being careful not to drag any point past, or on top of any other point. Repeat this process to add rows to your table.**

3. **You will need 10 rows of data. Be sure you have some small angle measures and some large angle measures. The angle measures should be greater than 0° and less than 90°.**

4. **Record the data from the computer in the table below.**

$m\angle MQN$	$m\overset{\frown}{MN}$	$m\overset{\frown}{PO}$	$m\overset{\frown}{MN} - m\overset{\frown}{PO}$
26.24	*75.45*	*22.97*	*52.48*
29.84	*85.92*	*26.24*	*59.68*
35.90	*99.89*	*28.09*	*71.80*
40.58	*113.21*	*32.05*	*81.16*
46.22	*130.52*	*38.09*	*92.43*
50.68	*143.71*	*42.35*	*101.36*
55.99	*163.39*	*51.40*	*111.99*
58.91	*172.42*	*54.60*	*117.82*
64.63	*192.27*	*63.01*	*129.25*
73.05	*241.94*	*95.84*	*146.10*

5. **What patterns do you observe in the table?**
Answers may vary. Students should observe the measure of the angle is one-half the difference of the measures of the intercepted arcs.

6. **To explore the relationship between the difference of the measures of the intercepted arcs and the measure of $\angle MQN$, transfer the necessary data from the table in question 4 to the table below.**

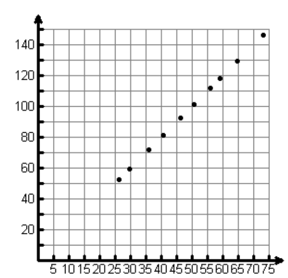
$m\angle MQN$ (x)	PROCESS	$m\overparen{MN} - m\overparen{PO}$ (y)
26.24	*(2) 26.24*	*52.48*
29.84	*(2) 29.84*	*59.68*
35.90	*(2) 35.90*	*71.80*
40.58	*(2) 40.58*	*81.16*
46.22	*(2) 46.22*	*92.43*
50.68	*(2) 50.68*	*101.36*
55.99	*(2) 55.99*	*111.99*
58.91	*(2) 58.91*	*117.82*
64.63	*(2) 64.63*	*129.25*
73.05	*(2) 73.05*	*146.10*
x	$2x$	y

7. **Use the process column to develop an algebraic rule that describes this relationship.**
$y = 2x$

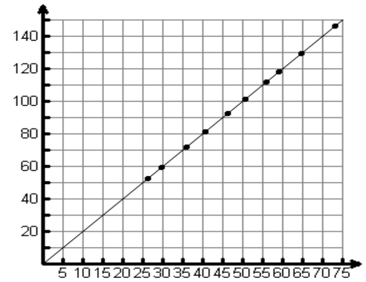
8. **Write a verbal description of the relationship between the difference of the measures of the intercepted arcs and the measure of the angle formed by the intersecting secants.**
Two times the measure of the angle is equal to the difference of the measures of the intercepted arcs. The difference of the measures of the intercepted arcs divided by 2 is equal to the measure of the angle.

9. **Create a scatterplot of difference of the arc measures vs. angle measure. Describe your viewing window.**

x-min = *0*
x-max = *75*
y-min = *0*
y-max = *150*



10. **Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.**



11. **Does the graph verify your function rule? Why or why not?**
Yes. The graph of the function rule passes through each data point.

12. **What is the measure of an angle formed by two intersecting secants if the measures of its intercepted arcs are 40° and 130°?**
45°

13. **What is the difference of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting secants 43°?**
86°

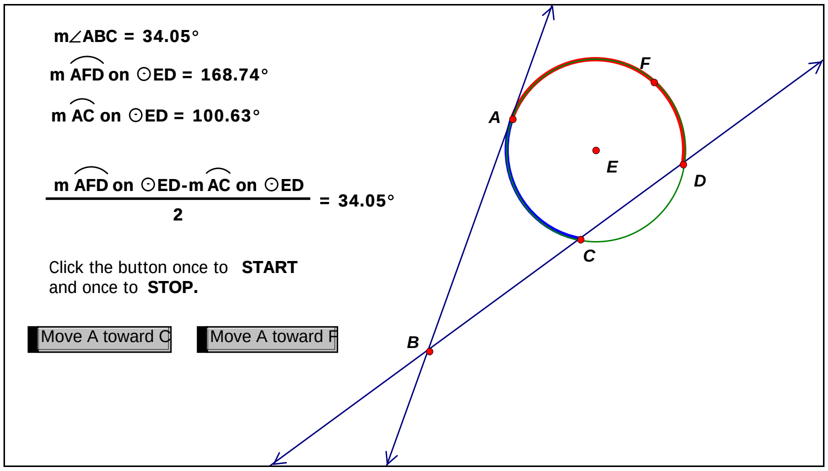
14. **Make a general statement about how you can determine the measure of the angle when you know the measures of the intercepted arcs.**
To determine the measure of the angle, subtract the measures of the two intercepted arcs then divide by 2.

15. **Make a general statement about how you can determine the difference of the measures of the intercepted arcs when you know the measure of the angle.**
To determine the difference of the measures of the intercepted arcs, multiply the measure of the angle by 2.

Other Intersecting Lines and Segments

1. Tangent and a Secant that intersect in the exterior of a circle

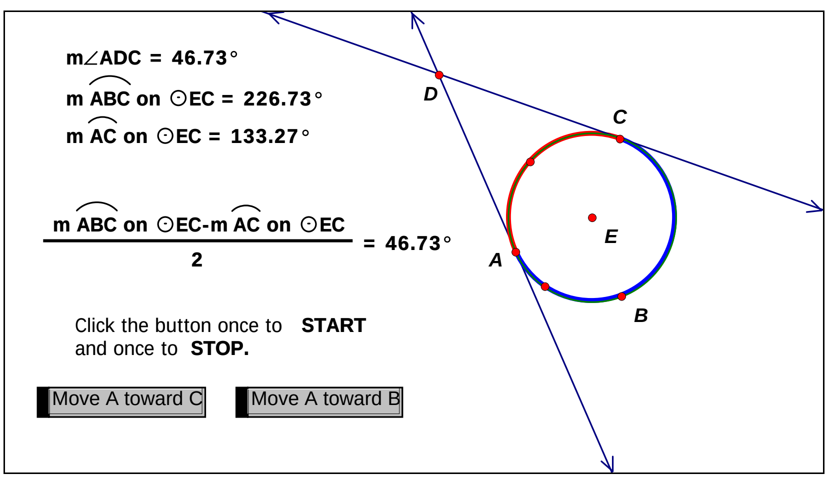
a. Open the sketch, "Tansecant-out.".



b. Click a button to move point A. What do you observe about the angle and arc relationships?
The measure of the angle is one-half the difference in the measures of the intercepted arcs.

2. Two tangents that intersect in the exterior of a circle

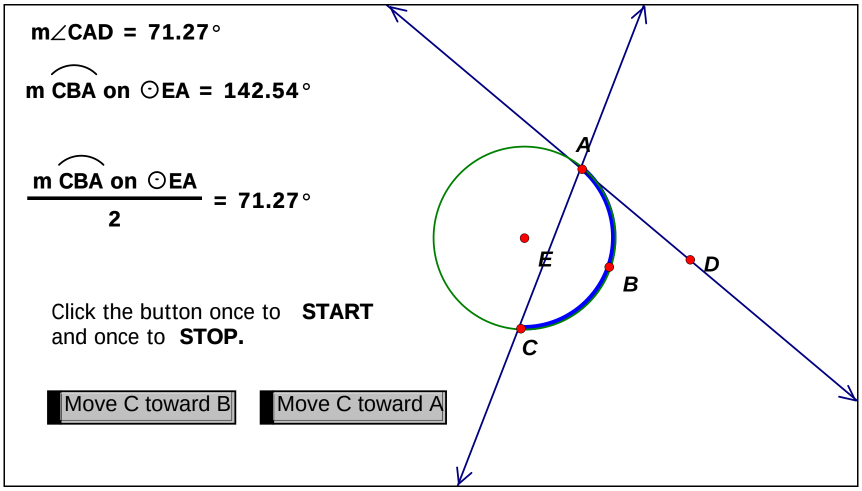
a. Open the sketch, "Twotangents-out."



b. Click a button to move point A. What do you observe about the angle and arc relationships?
The measure of the angle is one-half the difference in the measures of the intercepted arcs.

3. Tangent and a Secant that intersect on a circle

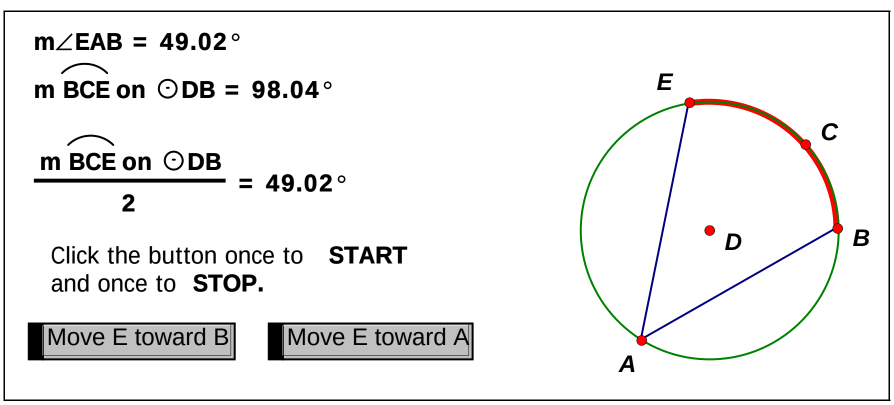
a. Open the sketch "Tansecant-on."



b. Click a button to move point C. What do you observe about the angle and arc relationships?
The measure of the angle is one-half the measure of the intercepted arc.

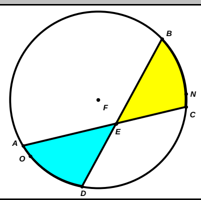
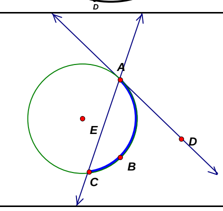
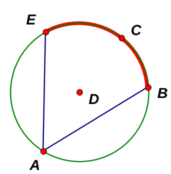
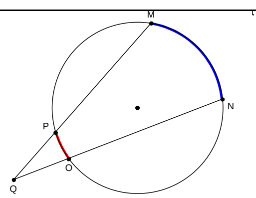
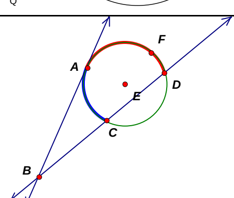
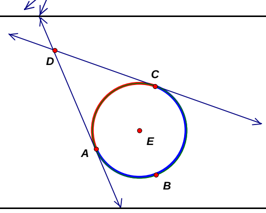
4. Two chords that intersect on a circle

a. Open the sketch "Twochords-on."



b. Click a button to move point *E*. What do you observe about the angle and arc relationships?
The measure of the angle is one-half the measure of the intercepted arc.

In the previous activities you investigated relationships among circles, arcs, chords, secants, and tangents. The vertex of the angle formed by the intersecting lines was either inside the circle, outside the circle or on the circle. Use what you discovered to complete the table below.

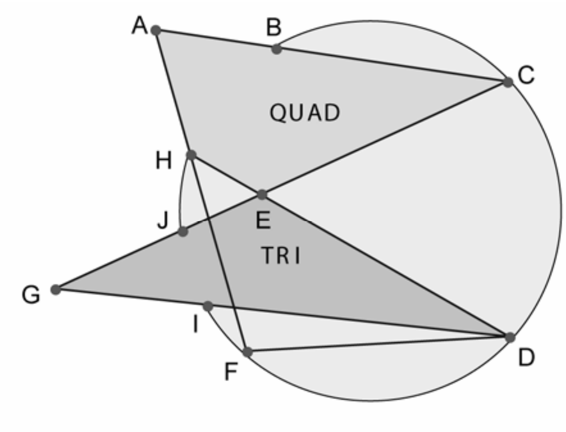
Diagram	Is the vertex of the angle inside, outside or on the circle?	How to calculate the measure of the angle
	*Inside the circle*	*The measure of the angle is one-half the sum of the measures of the intercepted arcs.*
	*On the circle*	*The measure of the angle is one-half the measure of the intercepted arc.*
	*On the circle*	
	*Outside the circle*	*The measure of the angle is one-half the difference in the measures of the intercepted arcs.*
	*Outside the circle*	
	*Outside the circle*	

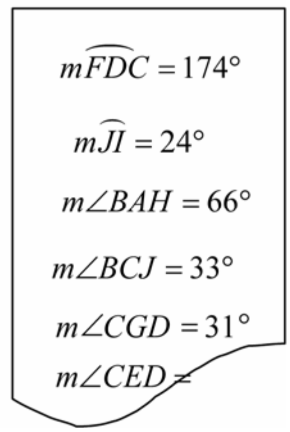
Complete the following generalizations about calculating angle measure.

1. When the vertex is **inside** the circle, _add_ the measures of the intercepted arcs then _divide by 2___.
2. When the vertex is **outside** the circle, *subtract the* measures of the intercepted arcs then __divide by 2.
3. When the vertex is **on** the circle, *divide the measure of the intercepted arc by 2* .

Quad-Tri Incorporated

The owners of Quad-Tri Inc. were in the process of designing a new emblem for their employee uniforms when a hurricane rolled in. After the hurricane, Pierre, the chief designer, could only find a torn sheet of paper that contained some of the measures he needed to complete the emblem. The design and the sheet of paper are shown below.





Pierre thinks the measure of angle CED must be 60°. Is he correct? Justify your answer.

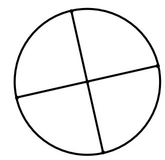
Answer: Pierre is not correct. Based on the known information, the measure of angle CED must be 55°.

Create an “Arc Measuring Tool”

1. You should have two sheets of Patty Paper. On each sheet construct a large circle. Be sure your circles are congruent to each other.

2. Cut out each circle and set one aside.

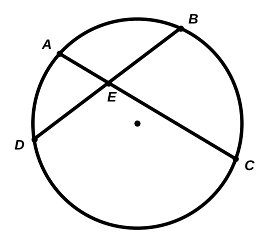
3. Fold a diameter in the second circle. Unfold the circle, then fold a second diameter perpendicular to the first diameter. You should have something that looks like this.



4. What special point is the point of intersection of the diameters? How do you know?

5. You now have a tool to estimate the number of degrees in arcs of your other circle. How can you make your “Arc Measuring Tool” a more precise measuring tool?

6. In your second circle, use a straight edge to draw two chords that intersect at a point that is not the center of the circle. Label your diagram as shown. Then use your available tools to find or estimate the necessary measures to complete the table below.



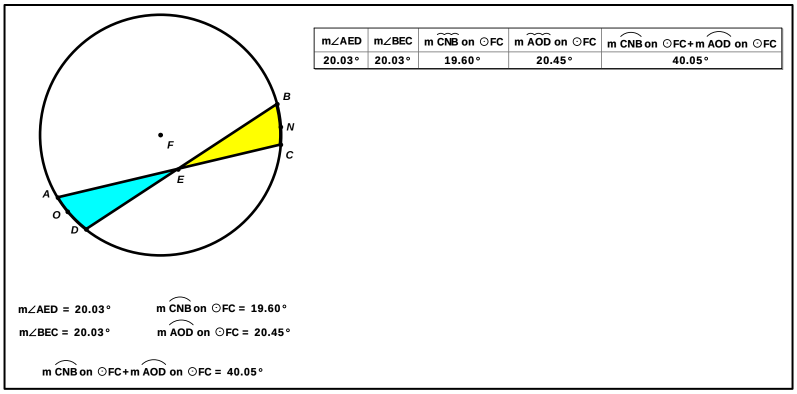
7. Record your name, your measurements and the name of each member of your group along with their measurements in the table.

Name	$m\angle AED$	$m\angle BEC$	$m\overset{\frown}{BC}$	$m\overset{\frown}{AD}$

8. What patterns do you observe in the table?

Angles Formed by Chords Intersecting Inside a Circle

Open the sketch **Twochords-in.**



1. Double click on the table to add another row, then click and drag point *B* away from point *N*. What do you observe?

2. Double click on the table again, and then move point *C* away from point *N*. Be sure point *N* stays between *B* and *C*.
3. Double click again, but this time drag point A away from point *O*. Double click again and drag point *D* away from point *O*. Be sure point *O* stays between *A* and *D*.
4. Be sure you have some small angle measures that are greater than 0° and some large angle measures that are less than 180°. Repeat this process until you have 10 rows in your table.
5. Record the data from the computer in the table below.

$m\angle AED$	$m\angle BEC$	$m\overset{\frown}{BC}$	$m\overset{\frown}{AD}$	$m\overset{\frown}{CNB} + m\overset{\frown}{AOD}$

6. What patterns do you observe in the table?

7. To explore the relationship between the sum of the measures of the intercepted arcs and the measure of $\angle AED$, transfer the necessary data from the table in question 3 to the table below.

$m\angle AED$ (x)	PROCESS	$m\overset{\frown}{CNB} + m\overset{\frown}{AOD}$ (y)
x		y

8. Use the process column to develop an algebraic rule that describes this relationship.

9. Write a verbal description of the relationship between the sum of the measures of the intercepted arcs and the measure of the angle formed by the intersecting chords.

10. Create a scatterplot of sum of the arc measures versus angle measure. Describe your viewing window and sketch your graph.

x-min =
x-max =
y-min =
y-max =

11. Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.

12. Does the graph verify your function rule? Why or why not?

13. What is the measure of an angle formed by two intersecting chords if the measures of its intercepted arcs are 30° and 120°?

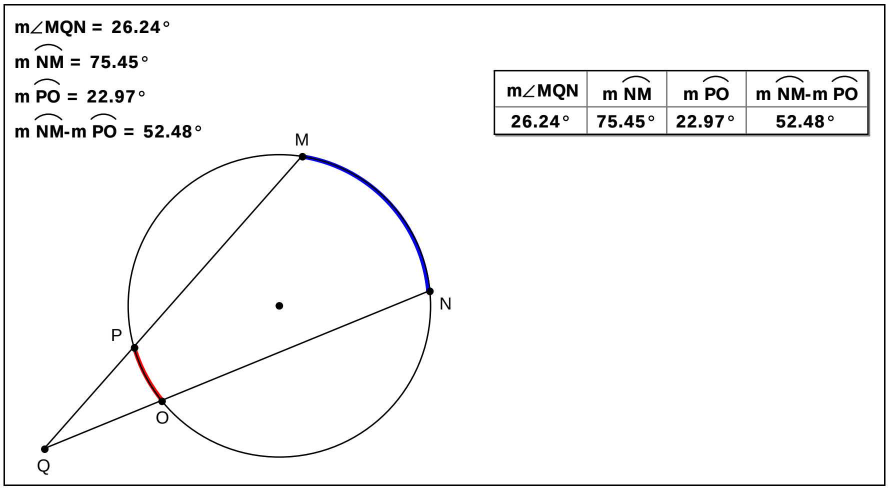
14. What is the sum of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting chords is 56°?

15. Make a general statement about how you can determine the measure of an angle formed by two intersecting chords when you know the measures of the intercepted arcs.

16. Make a general statement about how you can determine the sum of the measures of the intercepted arcs when you know the measure of the angle formed by two intersecting chords.

Angles Formed by Secants Intersecting Outside a Circle

Open the sketch **Twosecant-out**.



1. Double click on the table to add another row, then click and drag point *M*. What do you observe?

2. Double click on the table to add another row, and then move point *M* again. Double click again, but this time drag point N being careful not to drag any point past, or on top of any other point. Repeat this process to add rows to your table.

3. You will need 10 rows of data. Be sure you have some small angle measures and some large angle measures. The angle measures should be greater than 0° and less than 90°.

4. Record the data from the computer in the table below.

$m\angle MQN$	$m\widehat{MN}$	$m\widehat{PO}$	$m\widehat{MN} - m\widehat{PO}$

5. What patterns do you observe in the table?

6. To explore the relationship between the difference of the measures of the intercepted arcs and the measure of $\angle MQN$, transfer the necessary data from the table in question 4 to the table below.

$m\angle MQN$ (x)	PROCESS	$m\overset{\frown}{MN} - m\overset{\frown}{PO}$ (y)
x		y

7. Use the process column to develop an algebraic rule that describes this relationship.

8. Write a verbal description of the relationship between the difference of the measures of the intercepted arcs and the measure of the angle formed by the intersecting secants.

9. Create a scatterplot of difference of the arc measures vs. angle measure. Describe your viewing window

x-min =
x-max =
y-min =
y-max =

10. Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.

11. Does the graph verify your function rule? Why or why not?

12. What is the measure of an angle formed by two intersecting secants if the measures of its intercepted arcs are 40° and 130°?

13. What is the difference of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting secants is 43°?

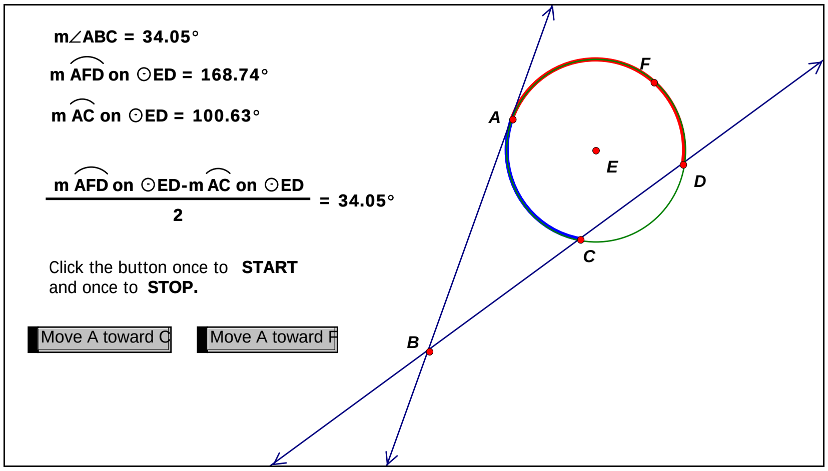
14. Make a general statement about how you can determine the measure of the angle when you know the measures of the intercepted arcs.

15. Make a general statement about how you can determine the difference of the measures of the intercepted arcs when you know the measure of the angle.

Other Intersecting Lines and Segments

1. Tangent and a Secant that intersect in the exterior of a circle

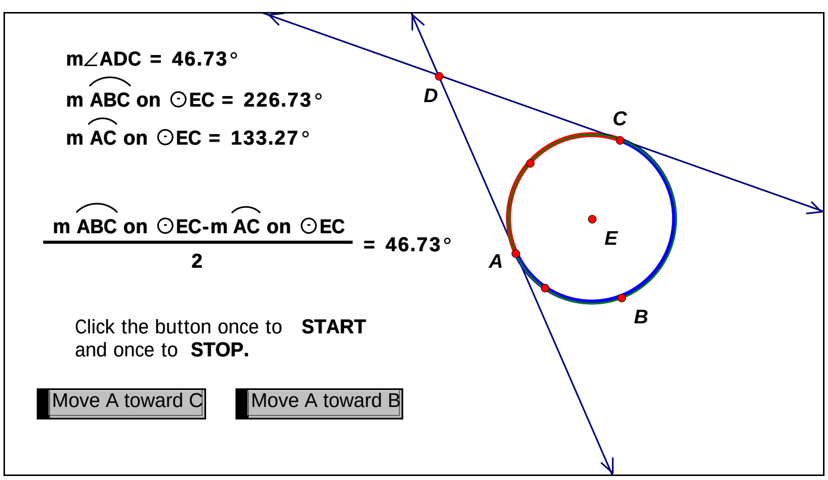
   a. Open the sketch, "**Tansecant-out.**"



   b. Click a button to move point A. What do you observe about the angle and arc relationships?

2. Two tangents that intersect in the exterior of a circle

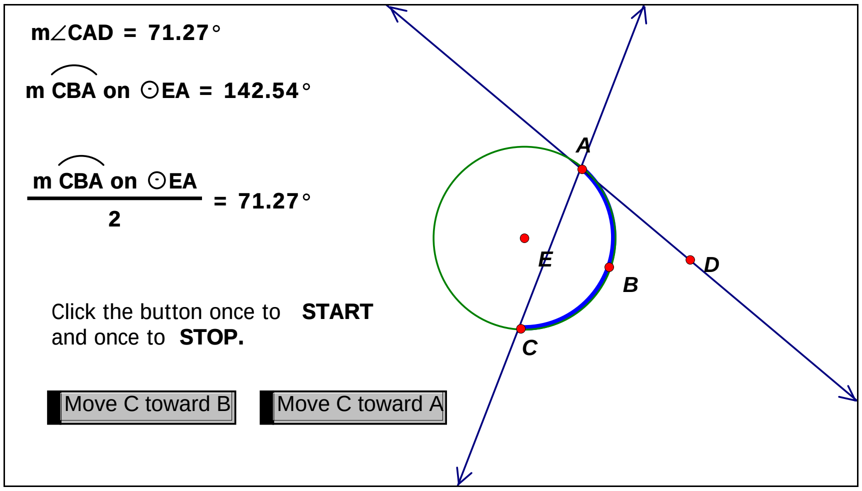
   a. Open the sketch, "**Twotangents-out.**"



   b. Click a button to move point A. What do you observe about the angle and arc relationships?

3. Tangent and a Secant that intersect on a circle

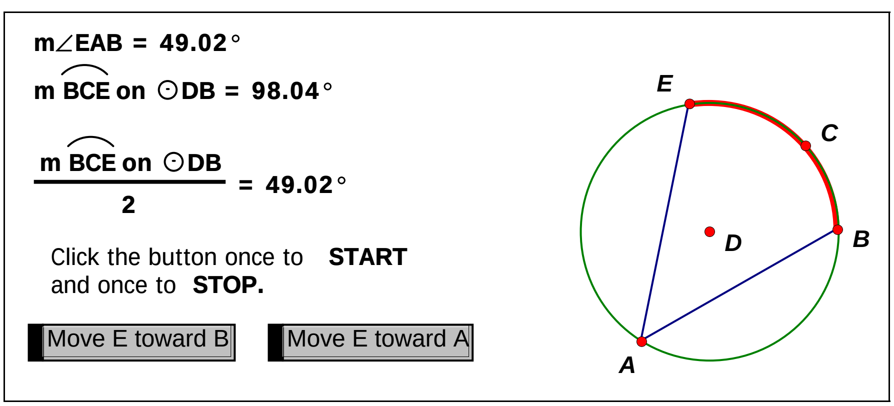
   a. Open the sketch "**Tansecant-on.**"



   b. Click a button to move point C. What do you observe about the angle and arc relationships?

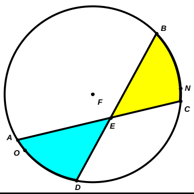
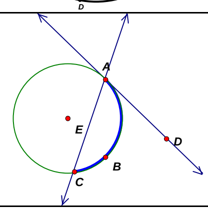
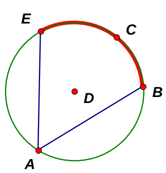
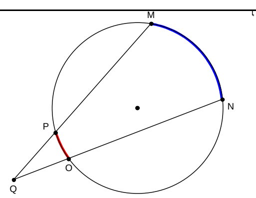
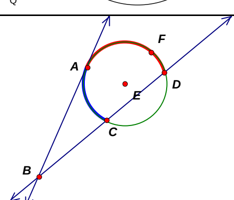
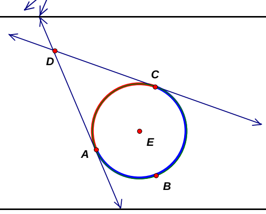
4. Two chords that intersect on a circle

   a. Open the sketch "**Twochords-on.**"



   b. Click a button to move point E. What do you observe about the angle and arc relationships?

In the previous activities you investigated relationships among circles, arcs, chords, secants, and tangents. The vertex of the angle formed by the intersecting lines was either inside the circle, outside the circle or on the circle. Use what you discovered to complete the table below.

Diagram	Is the vertex of the angle inside, outside or on the circle?	How to calculate the measure of the angle
		
		
		
		
		
		

Complete the following generalizations about calculating angle measure.

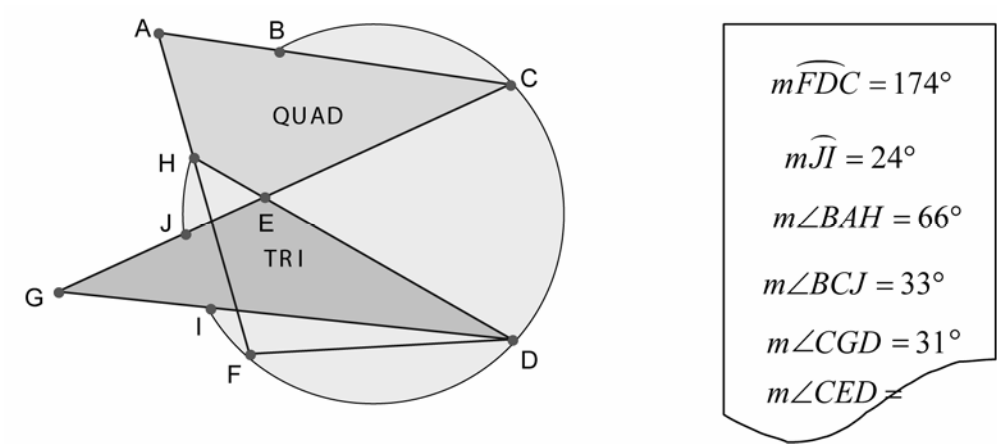
1. When the vertex is **inside** the circle, ______ the measures of the intercepted arcs then ________________.

2. When the vertex is **outside** the circle, ______ the measures of the intercepted arcs then ________________.

3. When the vertex is **on** the circle,_____________________________.

Quad-Tri Incorporated

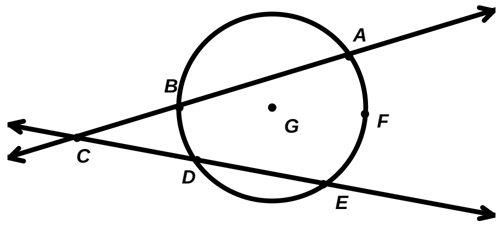
The owners of Quad-Tri Inc. were in the process of designing a new emblem for their employee uniforms when a hurricane rolled in. After the hurricane, Pierre, the chief designer, could only find a torn sheet of paper that contained some of the measures he needed to complete the emblem. The design and the sheet of paper are shown below.



Pierre thinks the measure of angle *CED* must be 60°. Is he correct? Justify your answer.

Circles, Angle Measures and Arcs

1 In the diagram $m\angle BCD = 25°$ and $m\overparen{BD} = 33°$.



Find $m\overparen{AFE}$.

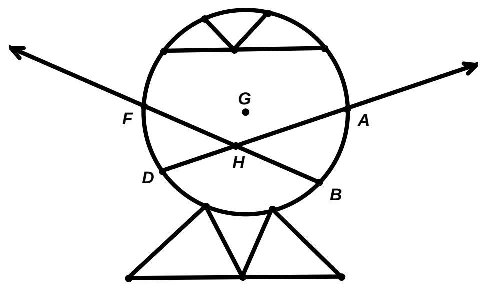
A 17°

B 50°

C 58°

D 83°

2 The metal sculpture shown was found in a recent archeological dig. $m\overparen{AB} = 46°$ and $m\overparen{FD} = 38°$



What is $m\angle DHB$?

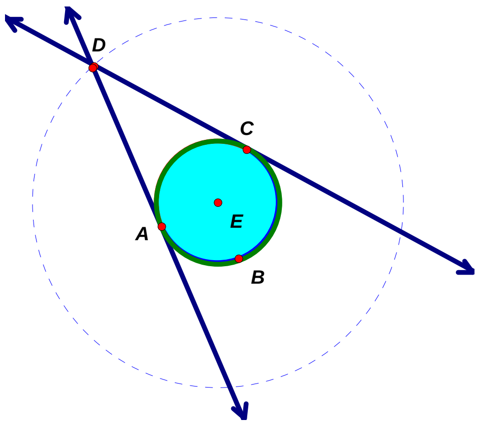
A 4°

B 42°

C 84°

D 138°

3 In the diagram, Point *D* represents a spacecraft as it orbits the Earth.



At this location 220° of the Earths surface is not visible from the spacecraft. What must be the $m\angle ADC$?

A 40°

B 80°

C 110°

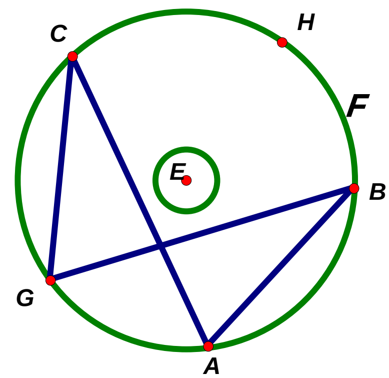
D 140°

4 Pablo created the sketch below.

$m\overset{\frown}{AB}$ on $\odot$EF = 80°

$m\overset{\frown}{CG}$ on $\odot$EF = 84°

$m\angle GBA$ = 31°



Based on the measurements he took, what must be $m\overset{\frown}{CHB}$?

A 134°

B 82°

C 67°

D 33.5°

Area of Regular Polygons

A.1B	Gather and record data and use data sets to determine functional relationships between quantities.
A.5C	Use, translate, and make connections among algebraic, tabular, graphical, or verbal descriptions of linear functions.
A.9A	Determine the domain and range for quadratic functions in given situations.
A.9D	Analyze graphs of quadratic functions and draw conclusions.
G.2A	Use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships.
G.2B	Make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.
G.3D	Use inductive reasoning to formulate a conjecture.
G.8A	Find areas of regular polygons, circles, and composite figures.
G.11C	Develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods.

Materials

Advance Preparation:

- Student access to computers with Geometer's Sketchpad and necessary sketches and/or a projection device to use Geometer's Sketchpad as a class demonstration tool.

For each student:

- Graphing calculator
- **Create an Area of Regular Polygons** activity sheet
- **Area of a Regular Hexagon versus the Length of its Apothem** activity sheet
- **Area of a Regular Pentagon versus the Length of its Apothem** activity sheet
- **Equilateral Triangles, Squares and Regular Octagons** activity sheet
- **Kick It Incorporated** activity sheet

For each student group of 3 - 4 students:

- Compasses
- Protractors
- Patty paper or tracing paper
- Rulers
- Scissors

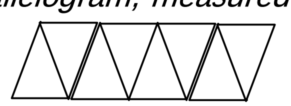
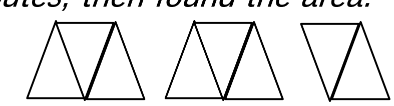
ENGAGE

The Engage portion of the lesson is designed to create student interest in the relationships between the area of regular polygons and the area of the triangles that compose them. This part of the lesson is designed for groups of three to four students.

1. Distribute a sheet of patty paper, a compass, ruler, protractor and a pair of scissors to each student.
2. Prompt students to use a compass to construct a large circle on the sheet of patty paper. Distribute the **Area of Regular Polygons** activity sheet. Students should follow the directions on the sheet.
3. Students will use paper folding to construct an octagon the cut it into 8 congruent triangles.
4. Students should use the available measuring tools to measure critical attributes, then calculate the area of the octagon.
5. Students will share their method of calculating the area with their group, and then with the whole class.
6. Debrief the activity using the Facilitation Questions.

Facilitation Questions – Engage Phase

1. When you folded your circle into 8 equal sectors, what was the measure of each central angle?
   45°, 360° divided by 8.
2. What do you observe about the 8 triangles that you cut out?
   Answers may vary. Students may observe that the triangles are congruent.
3. How do you know the triangles are congruent?
   Answers may vary. Students may stack the triangles or offer an informal proof using SAS since all radii and all central angles of the octagon are congruent.
4. How did you use your triangles to determine the area of your octagon?
   Answers may vary. Students should be able to explain their method. Students may have measured the base and height of one triangle, calculated the area of the triangle then multiplied it by 8. They may have arranged the triangles into the shape of a parallelogram or two trapezoids and a parallelogram, measured the critical attributes, then found the area.

   This activity sets the stage for exploring the relationship between the apothem of a regular polygon (height of the triangle) and the area of the polygon.
5. How can you determine a method that calculates the area of any regular polygon?
   Answers may vary. Students should realize that data for several polygons could be collected and analyzed to verify conjectures.

EXPLORE

The Explore portion of the lesson provides the student with an opportunity to participate actively in the exploration of the mathematical concepts addressed. This part of the lesson is designed for groups of three to four students.

1. Distribute the **Area of a Regular Hexagon versus the Length of its Apothem** activity sheet.
2. Students should open the sketch **HEXAGO.**
3. Have students follow the directions on the activity sheet to collect data and explore the relationship between the length of the apothem of a regular polygon and its area.
   Note: If students are not familiar with the operation of Geometer's Sketchpad, they will need the necessary instruction at this time. Also you may need to remind students to set their calculator MODE to Degrees,

Facilitation Questions – Explore Phase

1. What patterns do you notice in the table?
   Students should notice that as the apothem length increases, the area of the polygon increases, but not at a constant rate.

2. How do you know this is a non-linear relationship?
   Answers may vary. Students should notice that there is not a constant rate of change and that the graph is not linear.

3. In right triangle trigonometry, which trigonometric ratio should you use when you know the length of the leg adjacent to the reference angle and you want to find the length of the leg opposite the angle?
   The Tangent Ratio.

EXPLAIN

The teacher directs the Explain portion of the lesson to allow the students to formalize their understanding of the TEKS addressed in the lesson.

1. Debrief the **Area of a Regular Hexagon versus the Length of its Apothem** activity sheet. Use the Facilitation Questions to help students make connections among central angles, length of apothem, The Tangent Ratio, area of triangles and area of the polygon
2. Have each student group demonstrate how to use the graph and table features of the calculator to find the area given the apothem and find the apothem given the area.
3. Be sure students understand how to use the Geometer's Sketchpad sketches.

Facilitation Questions – Explain Phase

1. How did you determine the measure of the central angle of the hexagon?
   Divide 360° by 6'.
2. How did you determine the measure of the angle formed by the radius of the hexagon and its apothem?
   Divide the measure of the central angle by 2.
3. Why was the Tangent ratio used to find the measure of the leg of the right triangle?
   Tangent is used because the apothem is adjacent to the angle and the short leg of is opposite the angle.
4. Why was the length of the short leg of the right triangle multiplied by 2?
   Multiplying by 2 gives the length of the base of the triangle.
5. What is it about the relationship between apothem length and area that makes it a quadratic relationship?
   When a linear measure in a polygon is changed by a scale factor, the area of the polygon is changed by the square of that scale factor.
6. How did you determine the area of a hexagon with an apothem of 6.5?
   Students should be able to demonstrate how to use the graph and table features of the calculator to find the area.
7. How could you use your calculator to find the area of any regular hexagon when you know the length of the apothem?
   Students should be able to demonstrate how to use the graph and table features of the calculator to find the area of any regular hexagon.
8. How could you use your calculator to find the apothem of any regular hexagon when you know the area?
   Students should be able to demonstrate how to use the graph and table features of the calculator to find the apothem of any regular hexagon.

ELABORATE

The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS to a new situation. This part of the lesson is designed for groups of three to four students.

1. Distribute the **Area of a Regular Pentagon versus the Length of its Apothem** activity sheet.
2. Students should open the sketch **PENTA.**
3. Have students follow the directions on the activity sheet to collect data and explore the relationship between apothem length and area of a pentagon.
4. Debrief the **Area of a Regular Pentagon versus the Length of its Apothem** activity sheet.
5. Distribute the **Equilateral Triangles and Regular Octagons** activity sheet.
6. Prompt students to open the sketches as directed and explore the relationships.
7. Debrief the **Equilateral Triangles and Regular Octagons** activity sheet.

Facilitation Questions – Elaborate Phase

1. How did you determine the area of a pentagon with an apothem of 8.5?
   Students should be able to demonstrate how to use the graph and table features of the calculator to find the area.
2. How could you use your calculator to find the area of any regular pentagon when you know the length of the apothem?
   Students should be able to demonstrate how to use the graph and table features of the calculator to find the area of any regular pentagon.
3. How could you use your calculator to find the apothem of any regular pentagon when you know the area?
   Students should be able to demonstrate how to use the graph and table features of the calculator to find the apothem of any regular hexagon.
4. What would be the function rule for determining the area of a 12-sided polygon given its apothem?
   $y = 12x^2(tan(15))$
5. Explain how to find the area of any regular polygon when you know the length of its apothem.
   First find half the measure of the central angle. Find the tangent of that measure, then multiply by the number of sides and the square of the length of the apothem.

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.

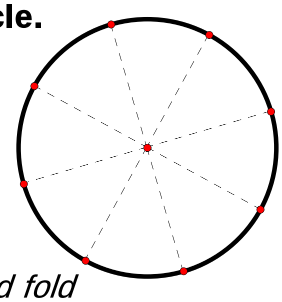
1. Provide each student with the **Kick It Incorporated** activity sheet.
2. Upon completion of the activity sheet, a rubric should be used to assess student understanding of the concepts addressed in the lesson.

Answers and Error Analysis for selected response questions:

Question Number	*TEKS*	*Correct Answer*	*Conceptual Error*	*Conceptual Error*	*Procedural Error*	*Procedural Error*	*Guess*
1	G.8(A)	B	D		A	C	
2	G.8(A)	C	A	B	D		
3	G.8(A)	B	A	C			D
4	G.8(A)	D	C		A	B	

Area of Regular Polygons

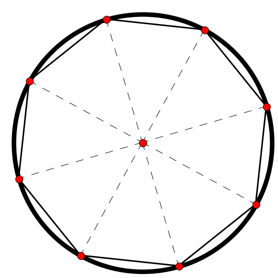
1. **On a sheet of patty paper construct a large circle.**



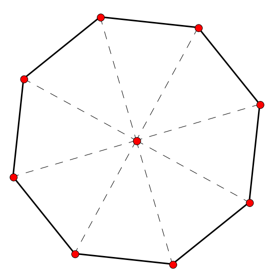
2. **Cut out the circle.**

3. **Use paper folding to divide the circle into 8 congruent sectors.**
   Students should fold a diameter, then fold a second diameter perpendicular to the first. Next, they should fold the bisectors of the 90° angles

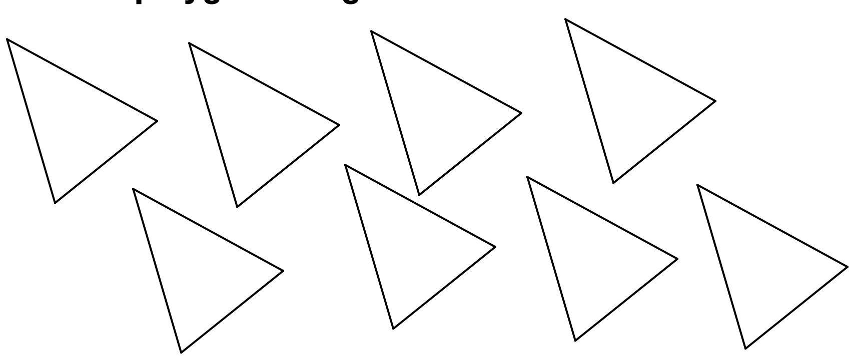
4. **Use a straight edge to connect the endpoints of the folded radii.**



5. **Cut out the polygon.**



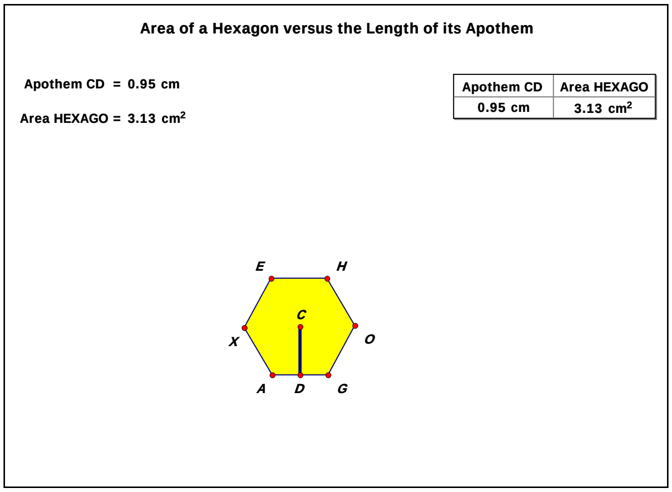
6. **Cut the polygon along each fold.**



7. **Determine the area of your original polygon.**
   Students may arrange the triangles to form familiar polygons, take measurements, then calculate the area.

Area of a Regular Hexagon versus the Length of its Apothem

Open the sketch **HEXAGO.**



1. **Double click on the table to add another row, then click and drag point *G* a short distance to the right. What do you observe?**
   The measures change.
2. **Double click on the table again, then move point *G* a little farther to the right. Repeat this process until you have 10 rows in your table. Keep the range of the apothem values between 0 and 12.**
3. **Record the data from the computer into the table below.**

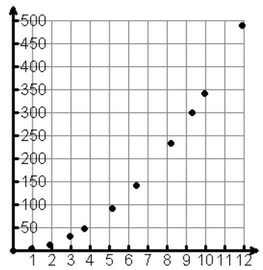
Apothem CD	*Area HEXAGO*
0.95	*3.13*
1.89	*12.32*
2.97	*30.51*
3.70	*47.44*
5.17	*92.48*
6.41	*142.49*
8.19	*232.45*
9.29	*299.06*
9.93	*341.79*
11.89	*490.10*

4. **What patterns do you observe in the table?**
   Answers may vary. Students should observe that as the length of the apothem increases the area increases, but not at a constant rate.

5. What is a reasonable domain and range for your data?
A reasonable domain includes values between 0 and 12. A reasonable range includes values between 0 and 500.

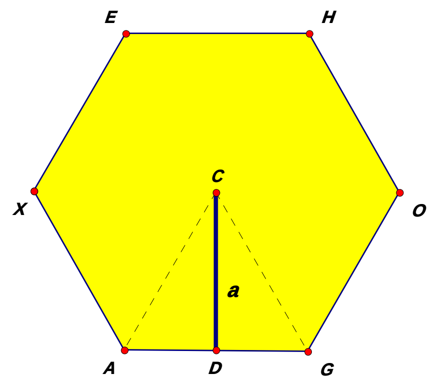
6. Create a scatterplot of Area of a Regular Hexagon versus the Length of its Apothem. Describe your viewing window and sketch your graph.

x-min = *0*
x-max = *12*
y-min = *0*
y-max = *500*



7. What observations can you make about your graph?
The graph appears to be a non-linear function, possibly quadratic.

8. To help develop a function rule for this situation use Hexagon *HEXAGO* to complete the following.



a. Since *HEXAGO* is a regular hexagon, $m\angle ACG = 60°$. What is $m\angle ACD$? *30°*

b. Using $\angle ACD$ as the reference angle, the trigonometric ratio "Tangent" can be used to find *AD* in terms of the apothem length, (*a*).

$$\tan 30° = \frac{AD}{a} \text{ or } AD = a(\tan 30°)$$

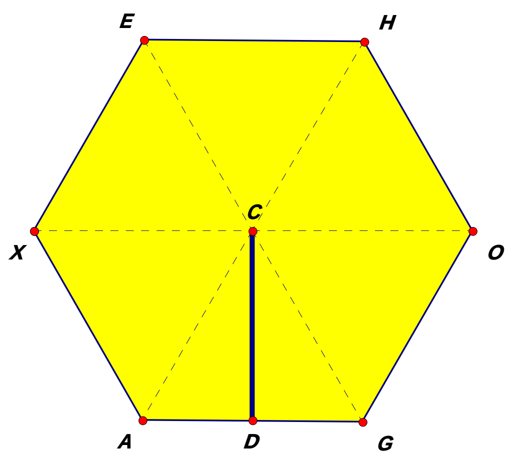
c. Write an expression for *AG* in terms of *a* and tan 30°.
Since AG = 2(AD) then AG = 2a tan 30°.

d. Recall the formula for area of a triangle, $Area = \dfrac{bh}{2}$. Using the length of the apothem (*a*) and your answer to question (c) above, write and simplify an expression for the area of $\triangle ACG$.

$$Area = \frac{2a(tan30°)a}{2}$$
$$Area = \frac{2a^2(tan30°)}{2}$$
$$Area = a^2(tan30°)$$

e. Draw the radius to each vertex of Hexagon *HEXAGO*. How many congruent isosceles triangles are formed?

6



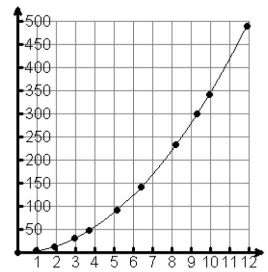
f. Use your answer to questions (d) and (e) above to write an expression for the area of a hexagon.

$$Area = 6a^2(tan30°)$$

g. Write your expression as a function rule that can be entered into the function graph tool of your graphing calculator.

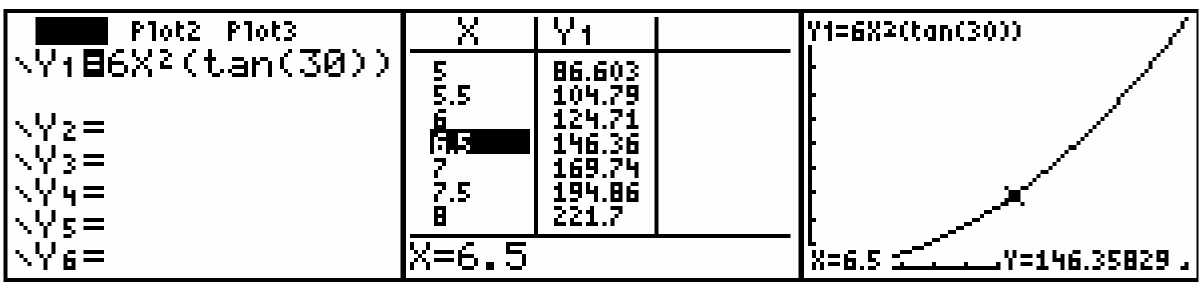
$$y = 6x^2(tan(30))$$

9. Enter your function rule into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

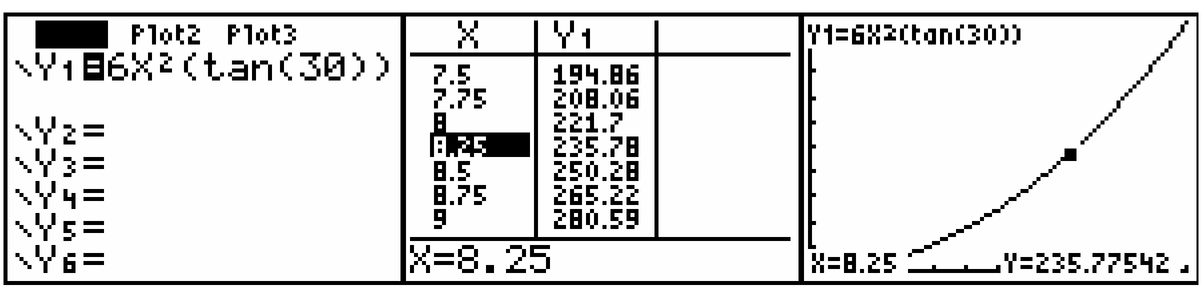


10. Does the graph verify your function rule? Why or why not?
Yes. The graph of the function rule passes through each data point.

11. Use your function rule and the graph and table features of your graphing calculator to determine the approximate area of a regular hexagon with an apothem of 6.5 centimeters.
Area ≈146.36 square centimeters.

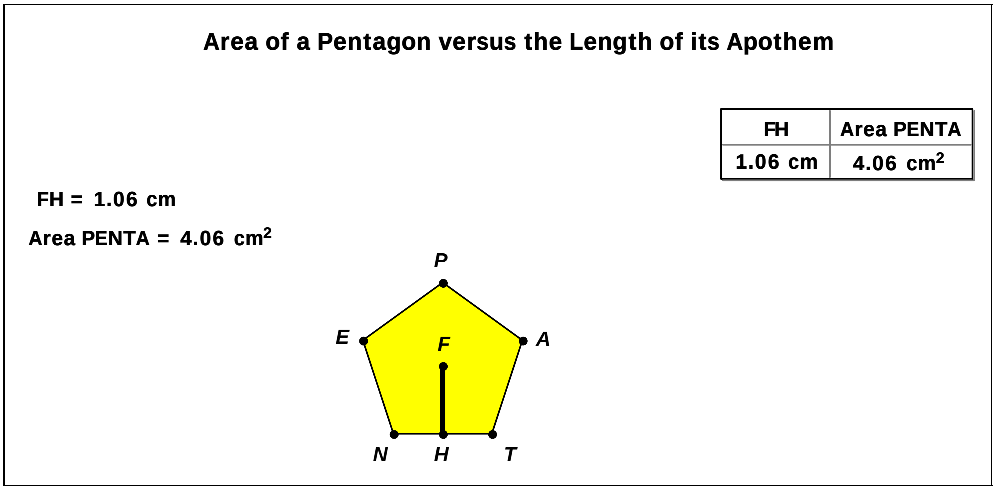


12. Use your function rule and the graph and table features of your graphing calculator to determine the approximate length of the apothem of a regular hexagon with an area of 235.78 square centimeters.
Apothem ≈8.25 centimeters.



Area of a Regular Pentagon versus the Length of its Apothem

Open the sketch **PENTA.**



1. **Double click on the table to add another row then click and drag point *T* a short distance to the right. What do you observe?**
   The measures change.
2. Double click on the table again, and then move point *T* a little farther to the right. Repeat this process until you have 10 rows in your table. Keep the range of the apothem values between 0 and 12.
3. Record the data from the computer in the table below.

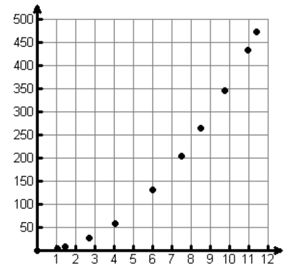
Apothem FH	*Area PENTA*
1.06	*4.06*
1.43	*7.39*
2.71	*26.63*
4.04	*59.20*
6.02	*131.53*
7.49	*204.06*
8.52	*263.79*
9.75	*345.27*
10.93	*434.23*
11.40	*472.22*

4. **What patterns do you observe in the table?**
   Answers may vary. Students should observe that as the length of the apothem increases the area increases, but not at a constant rate.

5. What is a reasonable domain and range for your data?
A reasonable domain includes values between 0 and 12. A reasonable range includes values between 0 and 500.

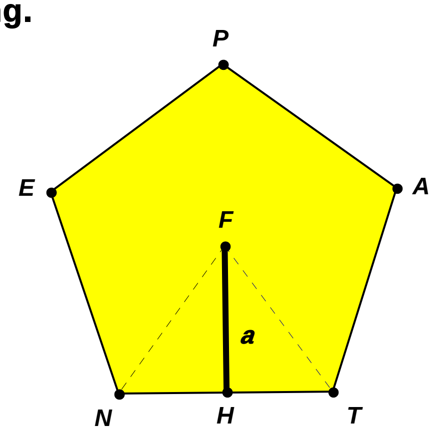
6. Create a scatterplot of Area of a Regular Pentagon versus the Length of its Apothem. Describe your viewing window and sketch your graph.

x-min = *0*
x-max = *12*
y-min = *0*
y-max = *500*



7. What observations can you make about your graph?
The graph appears to be a non-linear function, possibly quadratic.

8. To help develop a function rule for this situation use Pentagon *PENTA* to complete the following.



h. Since *PENTA* is a regular pentagon, what is $m\angle NFH$ **?** *36°*

i. Using $\angle NFH$ **as the reference angle, the trigonometric ratio, tangent, can be used to find *NH* in terms of the apothem length,** *a*.

j. Complete the expression $NH =$ __ $a(\tan 36°)$ __.

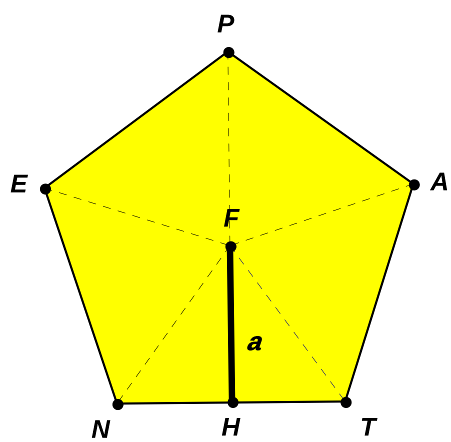
k. Write an expression for *NT* in terms of *a and* tan 36°.
Since NT = 2(NH) then NT= 2a tan 36°.

l. Recall the formula for area of a triangle, $Area = \frac{bh}{2}$**. Using the length of the apothem *a* and your answer to question (c) above, write and simplify an expression for the area of** ΔNFT **.**

$$Area = \frac{2a(tan36°)a}{2}$$
$$Area = \frac{2a^2(tan36°)}{2}$$
$$Area = a^2(tan36°)$$

m. Draw the radius to each vertex of Pentagon *PENTA.* How many congruent isosceles triangles are formed?

5



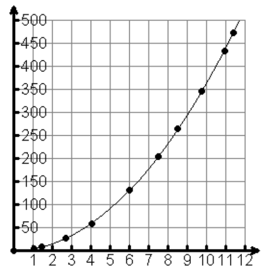
n. Use your answer to questions (d) and (e) above to write an expression for the area of a regular pentagon.

$Area = 5a^2(tan36°)$

o. Write your expression as a function rule that can be entered into the function graph tool of your graphing calculator.

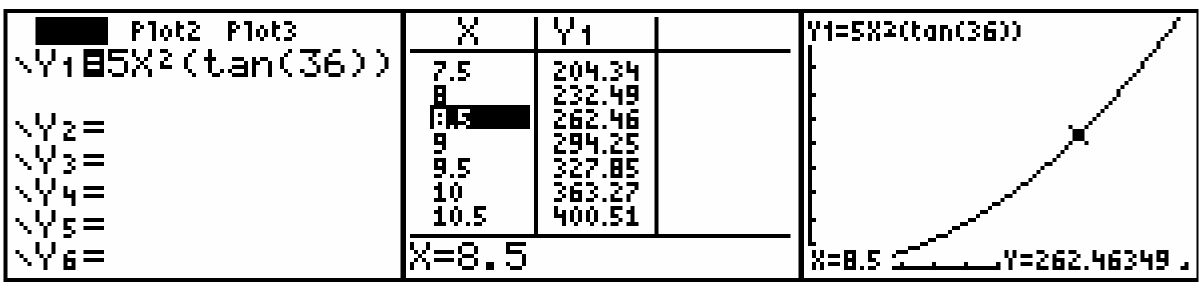
$y = 5x^2(tan(36))$

9. Enter your function rule into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

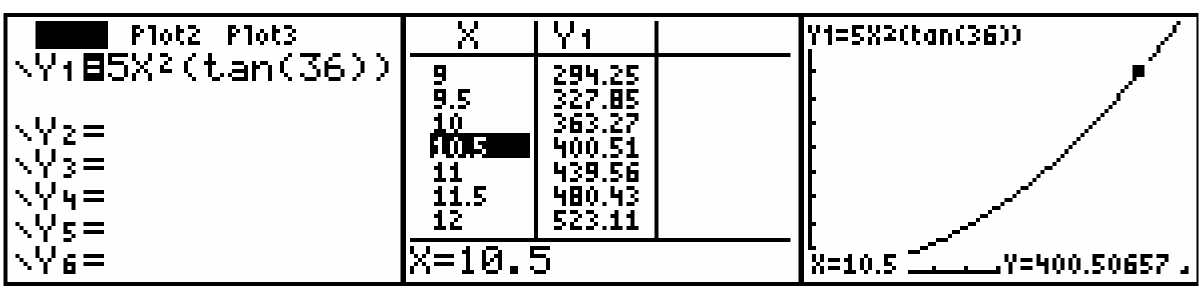


10. Does the graph verify your function rule? Why or why not?
Yes. The graph of the function rule passes through each data point.

11. Use your function rule and the graph and table features of your graphing calculator to determine the approximate area of a regular pentagon with an apothem of 8.5 centimeters. Sketch your graph and table.
Area ≈262.46 square centimeters.



12. Use your function rule and the graph and table features of your graphing calculator to determine the approximate length of the apothem of a regular pentagon with an area of 400.51 square centimeters. Sketch your graph and table.
Apothem ≈10.5 centimeters.



Equilateral Triangles and Regular Octagons

In the previous investigations you developed two function rules.

To determine the area, *y*, of a regular **hexagon** given the length of its apothem, *a*, the function rule is:

$$y = 6x^2(tan(30))$$

To determine the area, *y*, of a regular **pentagon** given the length of its apothem, *a*, the function rule is:

$$y = 5x^2(tan(36))$$

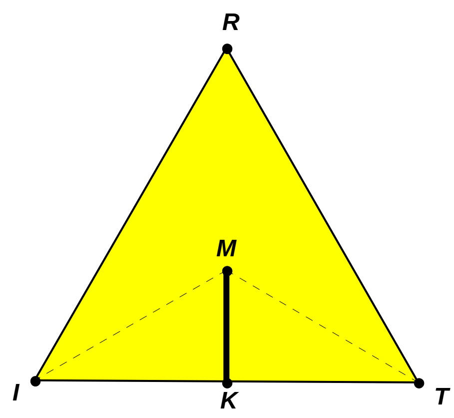
1. How are the function rules alike? What accounts for the similarities?

Both contain x^2 and tangent. The x^2 is because we multiply a·a in each case. In each case tangent used to find the length of the base of the triangle.

2. How are the function rules different? What accounts for the differences?

The rule for a hexagon has a 6 because a hexagon has 6 sides and tan 30 because one-half the measure of the central angle is 30°. The rule for a pentagon has a 5 because a pentagon has 5 sides and tan 36 because one-half the measure of the central angle is 36°.

3. Examine ΔTRI. What is $m\angle TMI$? *120°* **What is $m\angle IMK$?** *60°*



4. Based on your answers to questions 1, 2 and 3 above, write what you think will be the function rule to determine the area, *y*, of a regular triangle (equilateral) given the length of its apothem, *a*.

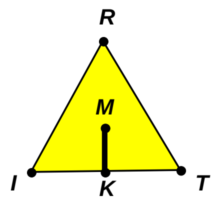
$y = 3x^2(tan(60))$

5. Open the sketch, "TRI."

Area of an Equilateral Triangle versus the Length of its Apothem

Apothem MK = 0.62 cm
Area △TRI = 2.01 cm²

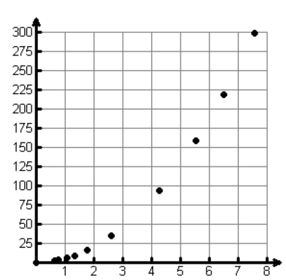
Apothem MK	Area △TRI
0.62 cm	2.01 cm²



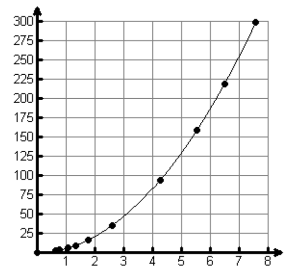
6. Click and drag point *T*. Double click on the table. Continue this process until you have at least 10 data points. Record your data in the table below.

Apothem MK	*Area Triangle TRI*
0.62	*2.01*
0.78	*3.13*
1.05	*5.73*
1.33	*9.16*
1.77	*16.20*
2.60	*35.19*
4.26	*94.13*
5.53	*159.08*
6.49	*218.62*
7.57	*298.13*

7. Create a scatterplot of Area of △*TRI* versus Apothem *MK*.

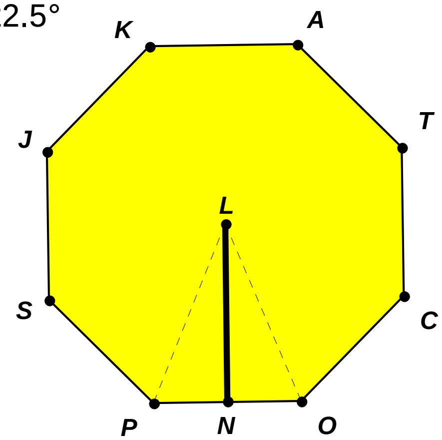


8. Enter your function rule from question 4 into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.



9. Does the graph verify your function rule? Why or why not?
Yes. The graph of the function rule passes through each data point.

10. Examine regular octagon *OCTAKJSP*. What is $m\angle PLO$ **?** *45°*
What is $m\angle PLN$ **?** 22.5°



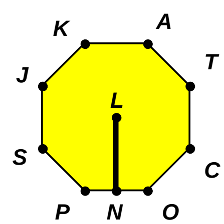
11. Write what you think will be the function rule to determine the area, *y*, of a regular octagon **given the length of its apothem, *a*.**
$y = 8x^2(tan(22.5))$

12. Open the sketch, "**OCTAGONS.**"

Area of a Regular Octagon versus the Length of its Apothem

Apothem LN = 1.06 cm

Area Octagon = 3.73 cm²

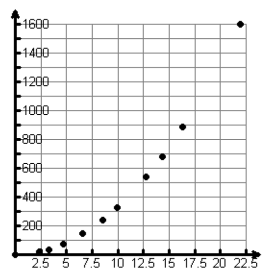


Apothem LN	Area Octagon
1.06 cm	3.73 cm²

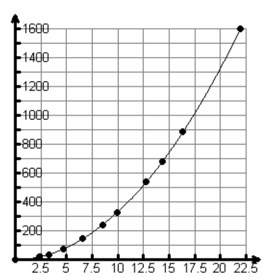
13. **Click and drag point *O*. Double click on the table. Continue this process until you have at least 10 data points. Record your data in the table below.**

Apothem LN	*Area Octagon*
2.32	*17.88*
3.25	*35.08*
4.71	*73.54*
6.59	*144.12*
8.52	*240.51*
9.91	*325.44*
12.73	*537.21*
14.31	*678.19*
16.33	*884.16*
21.97	*1598.84*

14. **Create a scatterplot of Area of the Octagon versus Apothem *LN*.**



15. **Enter your function rule from question 4 into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.**



16. **Does the graph verify your function rule? Why or why not?**
Yes. The graph of the function rule passes through each data point.

17. In the previous activities you investigated relationships between area of regular polygons and the length of their apothems. The table below includes function rules for triangles, pentagons, hexagons, and octagons. Fill in any missing information then develop a general function rule that can be used to find the area of any regular polygon.

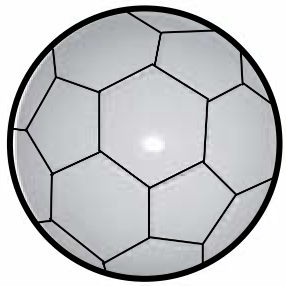
Regular Polygon	Number of Sides	Measure of the Central Angle	Function Rule
Triangle	*3*	*120°*	$y = 3x^2(tan(60))$
Square	*4*	*90°*	$y = 4x^2(tan(45))$
Pentagon	*5*	*72°*	$y = 5x^2(tan(36))$
Hexagon	*6*	*60°*	$y = 6x^2(tan(30))$
Heptagon	*7*	*51.43*	$y = 7x^2(tan(25.715))$
Octagon	*8*	*45°*	$y = 8x^2(tan(22.5))$
Any	*n*	$\frac{360}{n}$	$y = nx^2(tan\left(\frac{central\ angle}{2}\right))$

18. Use words to describe how to calculate the area of any regular polygon when you know the length of its apothem.

First find half the measure of the central angle. Find the tangent of that measure, then multiply by the number of sides and the square of the length of the apothem.

Kick It Incorporated

Banish's company, "Kick It Incorporated," manufactures soccer balls. To construct the covering for each ball 20 regular hexagons and 12 regular pentagons cut from synthetic leather are sewn together. The length of the apothem of each hexagon is 1.5 inches, and the length of the apothem of each pentagon is 1.2 inches.



The shipping manager needs to ship six inflated balls to a customer. He has a box with dimensions 22 inches by 15 inches by 8 inches. Can he fit 2 rows of 3 balls in the box? Justify your answer.

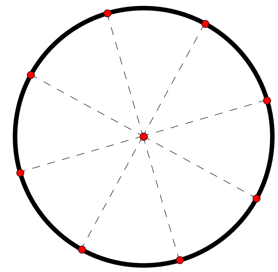
Answer: No. The surface area of a ball is approximately 208 square inches so the diameter of one ball is about 8 inches. Since the width of the box is only 15 inches, 2 balls will not fit. Since the length of the box is only 22 inches, 3 balls will not fit lengthwise. The height of the box may work, but the box could only hold 2 balls at most.

Area of Regular Polygons

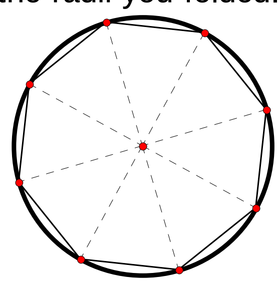
1. On a sheet of patty paper construct a large circle.

2. Cut out the circle.

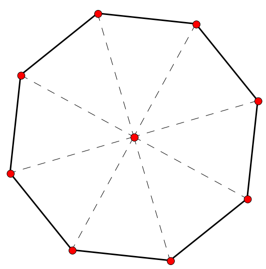
3. Use paper folding to divide the circle into 8 congruent sectors.



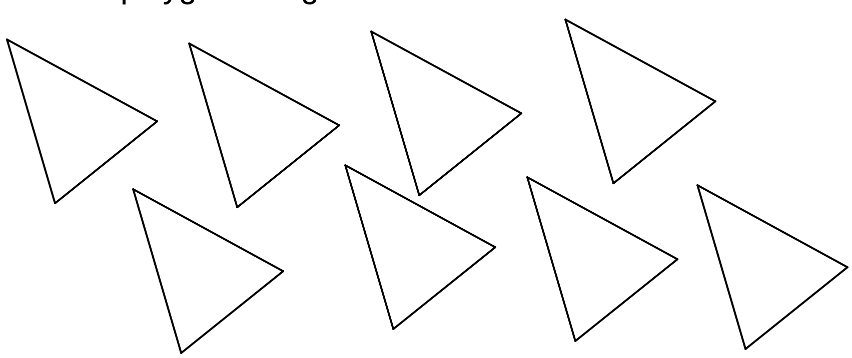
4. Use a straight edge to connect the endpoints of the radii you folded.



5. Cut out the polygon.



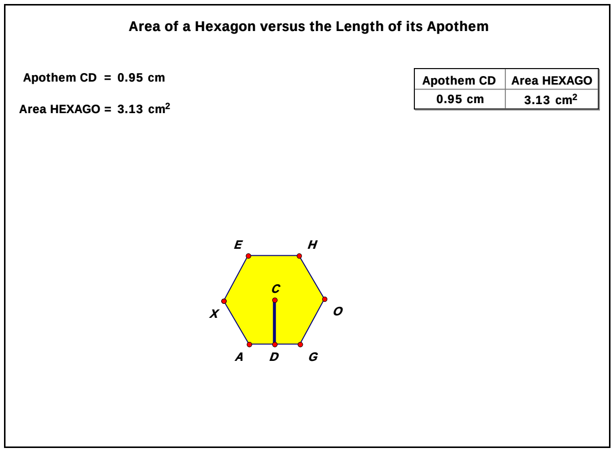
6. Cut the polygon along each fold.



7. Determine the area of your original polygon.

Area of a Regular Hexagon versus the Length of its Apothem

Open the sketch **HEXAGO.**



1. Double click on the table to add another row, then click and drag point *G* a short distance to the right. What do you observe?

2. Double click on the table again, then move point *G* a little farther to the right. Repeat this process until you have 10 rows in your table. Keep the range of the apothem values between 0 and 12.

3. Record the data from the computer into the table below.

Apothem CD	*Area HEXAGO*

4. What patterns do you observe in the table?

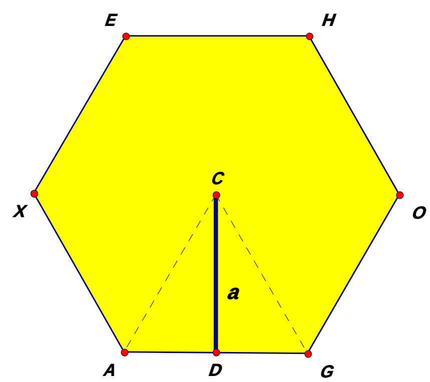
5. What is a reasonable domain and range for your data?

6. Create a scatterplot of Area of a Regular Hexagon versus the Length of its Apothem. Describe your viewing window and sketch your graph.

x-min =
x-max =
y-min =
y-max =

7. What observations can you make about your graph?

8. To help develop a function rule for this situation use Hexagon *HEXAGO* to complete the following.



a. Since *HEXAGO* is a regular hexagon, $m\angle ACG = 60°$. What is $m\angle ACD$?

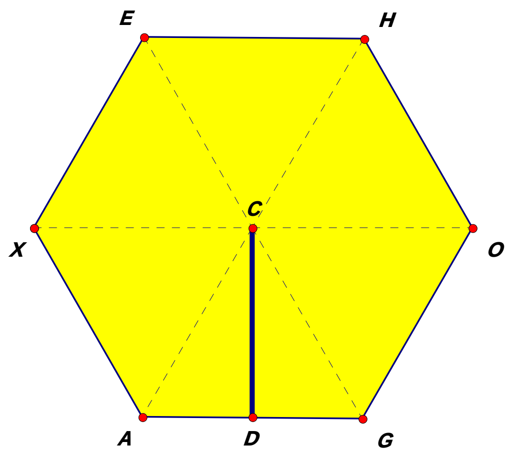
b. Using $\angle ACD$ as the reference angle, the trigonometric ratio "tangent" can be used to find *AD* in terms of the apothem length, *a*.

$$tan\,30° = \frac{AD}{a} \text{ or } AD = a(tan\,30°)$$

c. Write an expression for AG in terms of a and tan 30°.

d. Recall the formula for area of a triangle, $Area = \frac{bh}{2}$. Using the length of the apothem a and your answer to question (c) above, write and simplify an expression for the area of ΔACG.

e. Draw the radius to each vertex of Hexagon *HEXAGO*. How many congruent isosceles triangles are formed?



f. Use your answer to questions (d) and (e) above to write an expression for the area of a hexagon.

g. Write your expression as a function rule that can be entered into the function graph tool of your graphing calculator.

9. Enter your function rule into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

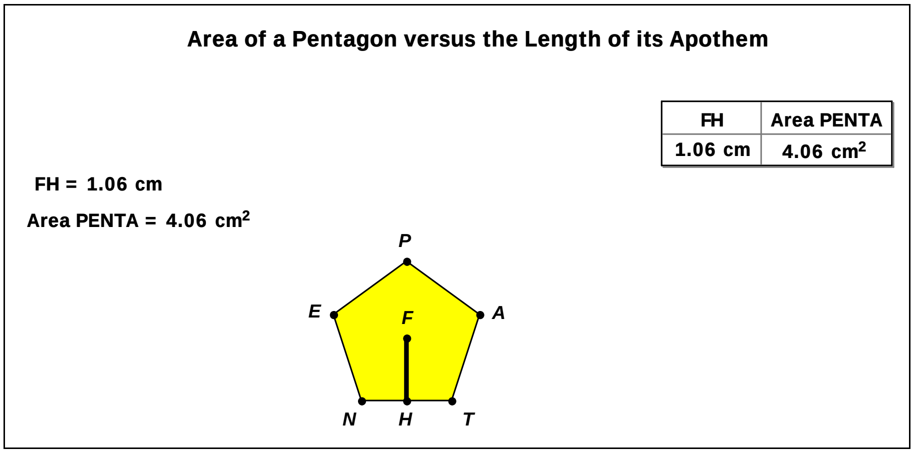
10. Does the graph verify your function rule? Why or why not?

11. Use your function rule and the graph and table features of your graphing calculator to determine the approximate area of a regular hexagon with an apothem of 6.5 centimeters. Sketch your graph and table.

12. Use your function rule and the graph and table features of your graphing calculator to determine the approximate length of the apothem of a regular hexagon with an area of 235.78 square centimeters. Sketch your graph and table.

Area of a Regular Pentagon versus the Length of its Apothem

Open the sketch **PENTA.**



1. Double click on the table to add another row, then click and drag point *T* a short distance to the right. What do you observe?

2. Double click on the table again, and then move point *T* a little farther to the right. Repeat this process until you have 10 rows in your table. Keep the range of the apothem values between 0 and 12.

3. Record the data from the computer in the table below.

Apothem FH	*Area PENTA*

4. What patterns do you observe in the table?

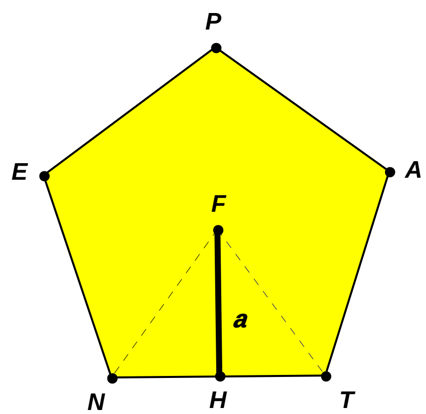
5. What is a reasonable domain and range for your data?

6. Create a scatterplot of Area of a Regular Pentagon versus the Length of its Apothem. Describe your viewing window and sketch your graph.

   x-min =
   x-max =
   y-min =
   y-max =

7. What observations can you make about your graph?

8. To help develop a function rule for this situation, use Pentagon *PENTA* to complete the following.



   a. Since *PENTA* is a regular pentagon, what is $m\angle NFH$?

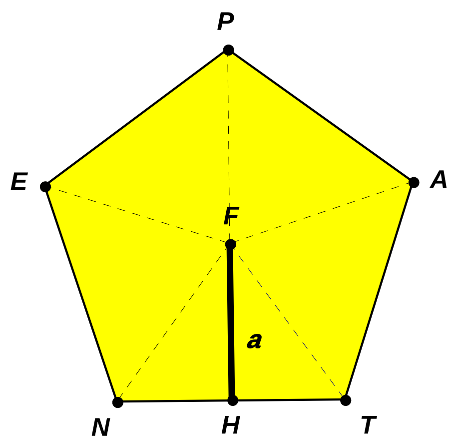
   b. Using $\angle NFH$ as the reference angle, the trigonometric ratio, tangent, can be used to find *NH* in terms of the apothem length, *a*.

   c. Complete the expression *NH =* ________________.

   d. Write an expression for *NT* in terms of, *a, and* tan 36°.

e. Recall the formula for area of a triangle, $Area = \frac{bh}{2}$. Using the length of the apothem, *a*, and your answer to question (c) above, write and simplify an expression for the area of ΔNFT .

f. Draw the radius to each vertex of Pentagon *PENTA.* How many congruent isosceles triangles are formed?



g. Use your answer to questions d and e above to write an expression for the area of a regular pentagon.

h. Write your expression as a function rule that can be entered into the function graph tool of your graphing calculator.

9. Enter your function rule into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

10. Does the graph verify your function rule? Why or why not?

11. Use your function rule and the graph and table features of your graphing calculator to determine the approximate area of a regular pentagon with an apothem of 8.5 centimeters. Sketch your graph and table.

12. Use your function rule and the graph and table features of your graphing calculator to determine the approximate length of the apothem of a regular pentagon with an area of 400.51 square centimeters. Sketch your graph and table.

Equilateral Triangles and Regular Octagons

In the previous investigations you developed two function rules.

To determine the area, *y*, of a regular **hexagon** given the length of its apothem, *a*, the function rule is:

$$y = 6x^2(tan(30))$$

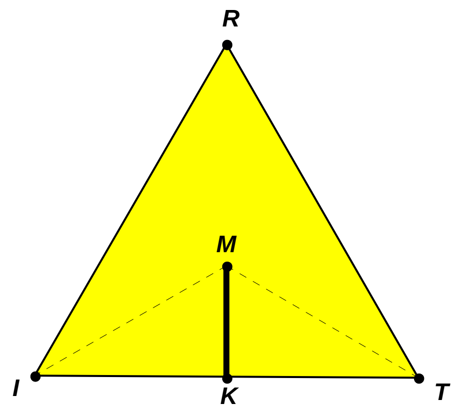
To determine the area, *y*, of a regular **pentagon** given the length of its apothem, *a*, the function rule is:

$$y = 5x^2(tan(36))$$

1. How are the function rules alike? What accounts for the similarities?

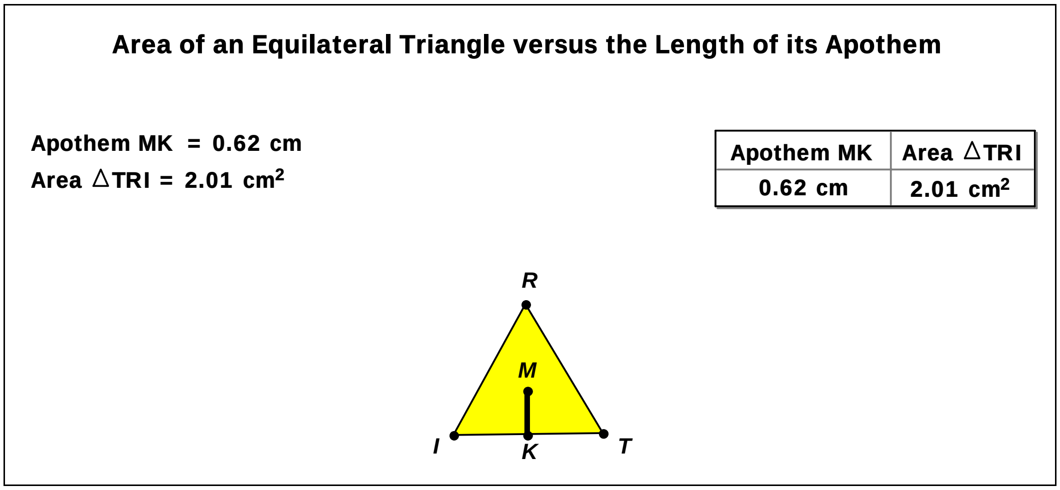
2. How are the function rules different? What accounts for the differences?

3. Examine ΔTRI. What is $m\angle TMI$? What is $m\angle IMK$?



4. Based on your answers to questions 1, 2 and 3 above, write what you think will be the function rule to determine the area, *y*, of a regular **triangle** (equilateral) given the length of its apothem, *a*.

5. Open the sketch, “**TRI.**”



6. Click and drag point *T*. Double click on the table. Continue this process until you have at least 10 data points. Record your data in the table below.

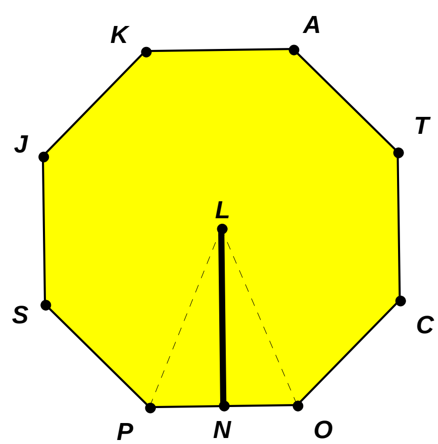
Apothem MK	*Area Triangle TRI*

7. Create a scatterplot of Area of $\triangle TRI$ versus Apothem *MK.*

8. Enter your function rule from question 4 into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

9. Does the graph verify your function rule? Why or why not?

10. Examine regular octagon *OCTAKJSP*. What is $m\angle PLO$? What is $m\angle PLN$?



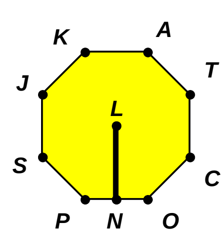
11. Write what you think will be the function rule to determine the area, *y*, of a regular **octagon** given the length of its apothem, *a*.

12. Open the sketch, "**OCTAGONS.**"

Area of a Regular Octagon versus the Length of its Apothem

Apothem LN = 1.06 cm

Area Octagon = 3.73 cm^2



Apothem LN	Area Octagon
1.06 cm	3.73 cm^2

13. Click and drag point *O*. Double click on the table. Continue this process until you have at least 10 data points. Record your data in the table below.

Apothem LN	*Area Octagon*

14. Create a scatterplot of Area of the Octagon versus Apothem *LN.*

15. Enter your function rule from question 4 into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

16. Does the graph verify your function rule? Why or why not?

17. In the previous activities you investigated relationship between area of regular polygons and the length of their apothems. The table below includes function rules for triangles, pentagons, hexagons, and octagons. Fill in any missing information, then develop a general function rule that can be used to find the area of any regular polygon.

Regular Polygon	Number of Sides	Measure of the Central Angle	Function Rule
Triangle	*3*	*120°*	$y = 3x^2(tan(60))$
Square			
Pentagon	*5*	*72°*	$y = 5x^2(tan(36))$
Hexagon	*6*	*60°*	$y = 6x^2(tan(30))$
Heptagon			
Octagon	*8*	*45°*	$y = 8x^2(tan(22.5))$
Any	*n*		

18. Use words to describe how to calculate the area of any regular polygon when you know the length of its apothem.

Kick It Incorporated

Banish's company, "Kick It Incorporated," manufactures soccer balls. To construct the covering for each ball 20 regular hexagons and 12 regular pentagons cut from synthetic leather are sewn together. The length of the apothem of each hexagon is 1.5 inches, and the length of the apothem of each pentagon is 1.2 inches.

The shipping manager needs to ship six inflated balls to a customer. He has a box with dimensions 22 inches by 15 inches by 8 inches. Can he fit 2 rows of 3 balls in the box? Justify your answer.

Composite Area

1 The floor of a room is in the shape of a regular hexagon. If the area of the room is 200 square feet, what is the approximate length of the apothem of the hexagon?

A 4.39 feet

B 7.60 feet

C 8.78 feet

D 38.11 feet

3 The table below was generated by a function rule that calculates the area of a regular polygon (*y*) given the length of its apothem (*x*).

X	Y1	
1	3.6327	
1.5	8.1736	
2	14.531	
2.5	22.704	
3	32.694	
3.5	44.501	
4	58.123	

X=1

Which polygon was it?

A triangle

B pentagon

C octagon

D decagon

2 The length of the apothem of the STOP sign at the corner of Ashcroft Drive and Ludington Street is 12 inches.

What is the area of the STOP sign?

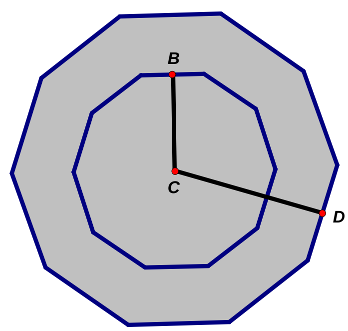
A 96

B 144 square inches

C 477.17

D 498.83

4 The drawing shows a cement walkway around a swimming pool. The walkway and the pool are in the shape of regular polygons. The length of $\overline{BC}$ is 18 feet and the length of $\overline{CD}$ is 29 feet.



What is the area of the walkway?

A 1052.7 square feet

B 2732.6 square feet

C 1873.2 square feet

D 1679.9 square feet